

Simulating One-Way ANOVA Using Resampling

Leslie Chandrakantha

lchandra@jjay.cuny.edu

Department of Mathematics and Computer Science

John Jay College of Criminal Justice of CUNY, New York, NY 10019, USA

Abstract

Analysis of Variance (ANOVA) is an important topic in introductory statistics. Many students struggle to understand the ANOVA concepts. In this paper, we describe how to use resampling with Excel Data Tables and standard functions to perform one-way ANOVA. We calculate different values of the F -statistic by resampling from the original sample and compute the p -value of the test. Using this approach, students will be able to get a better feel about the p -value concept. Our preliminary assessment shows that students have better feeling about this approach.

1. Introduction

Statistics is important not only in research but also in everyday usage. Statistical knowledge is relevant in many areas such as social science, natural science, engineering, and business. Therefore, students are required to take at least one course in statistics in many college degree programs. Understanding of statistical concepts such as sampling distributions, confidence intervals, hypothesis testing, and p -values is not easy for many students and many educators are conducting research on the teaching and learning of these concepts [2,10,15]. Computers have had a tremendous impact on teaching statistics at any level. Tishkovskaya et al. [16] have shown that the teaching and learning of statistics has benefited from the development of technological resources. The traditional way of teaching using books, lectures, and mathematical derivations does not give a good understanding of the concepts to many students at the introductory level. Some statistics instructors have been using computer simulation methods (CSM) in the classroom to teach difficult concepts. Computer simulation methods allow students to experiment with random samples for the purpose of clarifying abstract and difficult concepts of statistics. Cobb [5] noted that incorporating computer simulation to illustrate the key concepts and to allow students to discover important principles themselves enhances their knowledge. delMas et al. [6] evaluated the benefits of the computer simulation where students drew multiple samples from a variety of populations in order to observe the sampling distribution of the mean. Their evaluation of the simulation suggests that it provided an effective supplement to book and lecture-based methods of instruction. Johnson [9] used Minitab macros to resample in hypothesis testing. In his work, he pointed out that minimal or no mathematical skills are required and that the resampling approach helps students in understanding the statistical concepts. Black [1] was one of the first authors to recognize the usefulness of spreadsheets in a simulation context for teaching complex concepts of statistics. He used spreadsheet simulation to calculate the power of

the tests as a planning tool. Hagtvedt et al. [7, 8] have developed a Visual Basic Application (VBA) tool that is integrated into Excel spreadsheets to simulate sampling distributions and confidence intervals. Their assessment showed that the students who used their applications did significantly better than the students who did not use VBA based simulation. A comprehensive review of the literature of computer simulation methods used in all areas of statistics to help students understand difficult concepts has been done by Mills [11]. The results of the study of Mills [12] indicated some evidence that the computer simulation methods improve student understanding of abstract statistical concepts in introductory classes.

There are many on-line sources that allow students to simulate random variables and test statistics and to visualize their distributional properties. However, almost without exception, these on-line sources either build their simulation application using something other than a simple spreadsheet, or may not provide the source code. These web-based applications are written in languages such as Java, JavaScript, or some other programming languages, and their usefulness to introductory level students who are majoring in social science is somewhat questionable. Generally, introductory statistics students do not have the necessary skills to produce codes that would replicate what can be found in these on-line sites. Furthermore, web-based applications require an internet connection to be used in the classroom or anywhere else. An Excel demonstration can be used with or without an internet connection. As long as the students have access to a computer with Excel, they can perform these simulations and visualize the intended concepts.

In this paper, we describe how to use Excel standard formulas and the Data Table function to perform simulation using resampling in teaching one-way analysis of variance (ANOVA). ANOVA is one of many important topics in the introductory statistics curriculum. ANOVA is commonly used in many areas to compare means of several populations. The Rossman/Chance Applet Collection [14] includes a Java applet that simulates the ANOVA table for given sample sizes. Combination of Excel standard functions and Data Tables provides a convenient way to conduct simulation and resampling. Chandrakantha [3] used Excel Data Tables in teaching confidence intervals and hypothesis testing in introductory statistics classes. Christie [4] provides a way to use Excel Data Tables for estimating the population mean and the correlation.

This paper is organized as follows. In section 2, we give a brief overview of one-way ANOVA and an example. In section 3, the Data Tables and the resampling process of simulating ANOVA is introduced. In section 4, p -value calculations and the empirical distribution of the F -statistic are presented. In sections 5 and 6, a comparison of two teaching methods and concluding remarks are given.

2. Overview of ANOVA

The one-way analysis of variance procedure, referred to as ANOVA, is used to test the following hypothesis: the means of two or more populations are the same against the alternative that not all

population means are the same. The validity of the ANOVA test for equality of means requires following assumptions. The populations from which samples are drawn have the same variance, σ^2 . It also requires that each of the samples is obtained from a population with a normal distribution and each sample is randomly selected and is independent of all other samples.

The ANOVA test is performed by calculating two estimates of the variance, σ^2 , of the population distributions: the variance between samples and the variance within samples. The variance between samples is also known as mean square between samples (MSB) and the variance within samples is also known as mean square within samples (MSW or MSE). We will use the notation MSB and MSW here. Both MSB and MSW estimate the variance of populations, σ^2 . MSB is based on the values of the means of the samples taken from populations and MSW is based on the individual values in the samples. If the means of the populations under consideration are not equal, the variation among the means of respective samples is expected to be large, and, therefore, the value of MSB is expected to be large. Both MSB and MSW are obtained using sum of squares and hence each associates the degrees of freedom. Let's assume k is the number of populations under consideration and n is the number of data values in all samples. MSB measures the variation of the k sample means around the overall mean, and then its degrees freedom are $k - 1$. MSW measures the variation within k samples of n observations all together and its degrees of freedom are $n - k$.

The value of the F -statistic for the ANOVA test is calculated as

$$F = \frac{\text{Variance between samples}}{\text{Variance within samples}} = \frac{MSB}{MSW}.$$

This test statistic is assumed to be an F -statistic with degrees of freedom $k - 1$ and $n - k$ respectively. The formulas for calculating MSB and MSW can be found in any introductory statistics text. The one-way ANOVA test is always right-tailed and the p -value is computed using the right tail of the F distribution curve.

Example: The following example is used to demonstrate the simulation of ANOVA test to the class. A university employment office wants to compare the amount of time taken by graduates with three different majors to find their first full time job after graduation. *Table 2.1* lists the times (in days) for random samples of business, computer science, and engineering majors who graduated recently.

Business	208	162	240	180	148	312	176	292
Computer Science	136	113	281	128	305	147	232	
Engineering	126	275	363	146	298	392		

Table 2.1: Sample of times (in days)

Can we conclude that the mean time taken to find their first full time job for all graduates in these fields is the same?

The null and alternative hypotheses are defined as follows. In this example, we compare the mean time taken to find their first job for graduates in the three populations: Business, Computer Science, and Engineering. We assume the notation, μ_i represents the mean time taken to find the first job for the population i .

$H_0: \mu_1 = \mu_2 = \mu_3$. This means all population means are the same.

H_1 : Not all population means are the same.

Under the null hypothesis, the data comes from populations with the same mean and we are interested in how often the value of the test statistic is as large as the one we observe will occur by chance if, in fact, this hypothesis is true.

Now we show how to solve this problem traditionally. First, we verify the validity of the assumptions. Following Peck et al. [13], if sample sizes are so small, a separate boxplot for each sample is of little value in checking normality. In this case, a single combined plot can be constructed by first subtracting the sample mean from each value in the corresponding sample, and then constructing a normal probability or box plot for all n deviations from their respective means. The boxplot should be reasonably symmetric. *Figure 2.1* shows the combined boxplot for our example. This boxplot is roughly symmetric, and there are no outliers. For identical variance assumption, the ANOVA test can safely be used if the largest of the sample standard deviations is at most twice the smallest one. The largest standard deviation in our example is 109.90, which is only about 1.8 times the smallest standard deviation, 61.01. We can assume that each sample is randomly selected and is independent of others. The assumptions of ANOVA are reasonable.

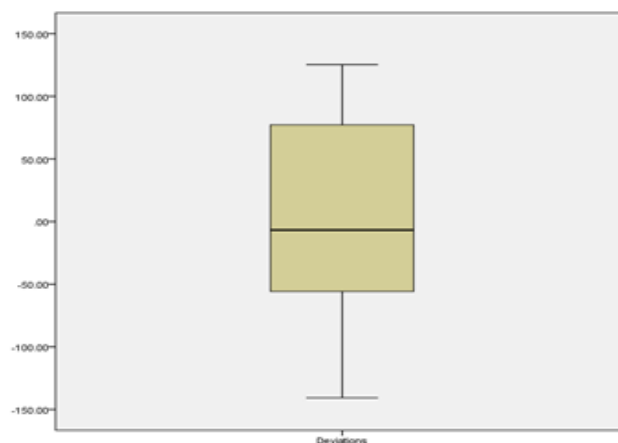


Figure 2.1: Boxplot of deviations from means

The F -statistic is given by the ratio of the variance between samples (MSB) and the variance within samples (MSW). The value of MSB is calculated by dividing the sum of squares between samples (SSB) by degrees of freedom between samples which are $k - 1$. The following formula is used to calculate MSB:

$$MSB = \frac{SSB}{k-1} = \frac{\left(\frac{T_1^2}{n_1} + \frac{T_2^2}{n_2} + \frac{T_3^2}{n_3} - \frac{T^2}{n} \right)}{k-1}$$

where T_i is the sum of the values in the sample from i^{th} population and T is the sum of the values in all samples. n_i is the number values in the sample from i^{th} population and n is the number of values in all samples.

The value of MSW is calculated by dividing the sum of squares within samples (SSW) by degrees of freedom within samples which are $n - k$. The following formula is used to calculate MSW:

$$MSW = \frac{SSW}{n-k} = \frac{\sum x_{ToT}^2 - \left(\frac{T_1^2}{n_1} + \frac{T_2^2}{n_2} + \frac{T_3^2}{n_3} \right)}{n-k}$$

where $\sum x_{ToT}^2$ is the sum of all data squares.

For our example, $T_1 = 1718$, $T_2 = 1362$, $T_3 = 1600$, $T = 4680$, $\sum x_{ToT}^2 = 1182958$, $n_1 = 8$, $n_2 = 7$, $n_3 = 6$, $n = 21$, and $k = 3$. Substituting these values into above two formulas, we get $MSB = 8821.01$ and $MSW = 6796.92$. Then the value of the F -statistic is $8821.01/6796.92 = 1.2978$. To find the approximate p -value for this F -statistic value, we use the F distribution table for F distribution with 2 and 18 degrees of freedom. The approximate p -value is larger than 0.10. This larger p -value allows us to conclude that the means of the three populations are the same. In the next section, we compute the value of the F -statistic for different resamples using Data Tables and the corresponding p -value. The visual display of the sampling distribution of the test statistic using simulation will help students to grasp the concept of the p -value.

3. Simulation Using Excel Data Tables

3.1 Data Table Introduction

Data Tables are part of a group of commands that are called the what-if analysis tools in Excel. When you use Data Tables, you are doing a what-if analysis. What-if analysis is the process of changing the values in cells to see how those changes will affect the outcome of formulas on the worksheet. We can use the Data Table function to compute the values of a test statistic for different random samples of data. The Data Table function is available in all Excel versions.

For example, a company’s projected profit may depend on number of units sold and some other factors. We can set up a spreadsheet using Data Tables to calculate the projected profit for different values of number of units sold. *Figure 3.1* shows a simple Data Table setup.

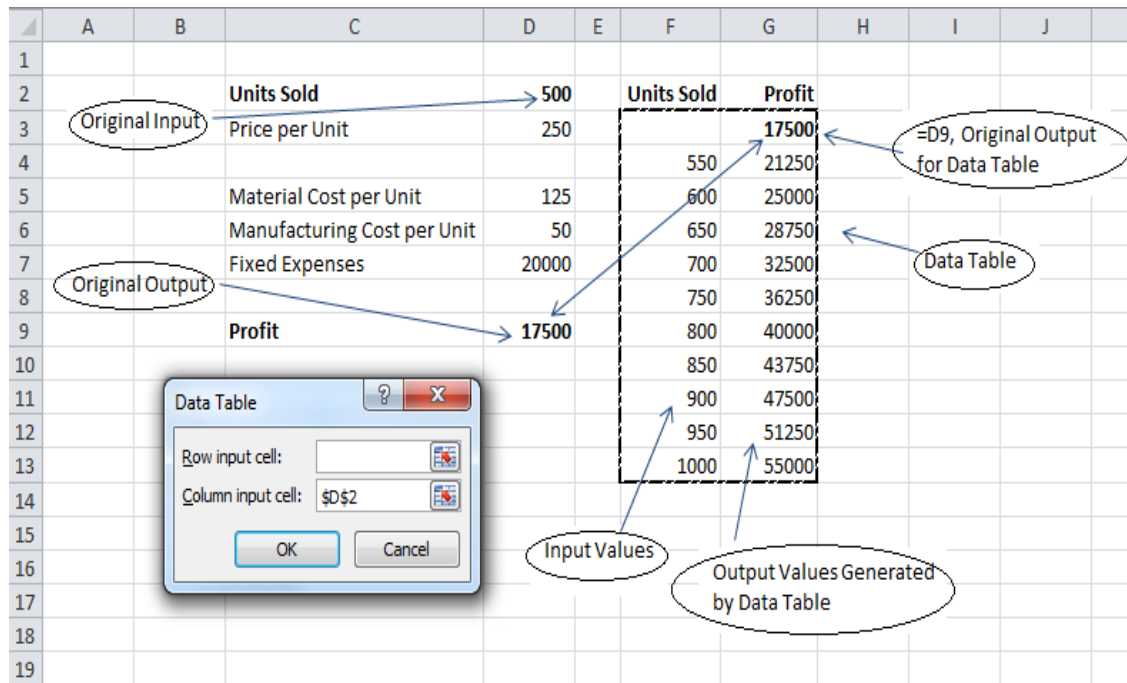


Figure 3.1: Data Table Setup

The original input is the number of units sold in cell *D2*. The original output is the profit in cell *D9* which is calculated based on the original input. The various possible values of input are put in the left column of the Data Table. The formula containing the original profit in cell *D9* is put at the top of the second column of the Data Table. We select the cell range *F3:G13* and select *Data > What IF Analysis > Data Tables*. This opens the Data Table Dialog Box shown in the *Figure 3.1*. We leave the Row input cell blank and put the original input’s cell reference, *D2*, for the Column input cell. Once we click OK, the Data Table tool runs down the left hand column of the table, substituting the value in that column into the original column input cell in *D2*. The formula at the top right column of the table is evaluated for each substitution of the input cell value to compute the profit values in the right column. This way is more convenient than changing the input (units sold) manually and recoding the profit for each input in a table.

To generate the values of a statistic for different samples using the Data Table, first we calculate the value of the statistic based on a random sample. This will be our original output value. Now we set up our Data Table by putting this value (formula for this output value) in the top cell of the right column. We leave the left column blank and select the table and access the Data Table dialog box using menu bar *Data > What If Analysis > Data Tables*. For the column input cell in the Data Table dialog box, we type an empty cell reference that has no part in this Data Table set up. Excel generates a new sample and computes the value of the statistic for each substitution of

this empty input cell and fills the table. Copying the formula down of the output cell does not work in this case. If we do this manually, we need to recalculate the statistic by repeated sampling by pressing F9 key and recording these values in the column. The Data Table function recalculates the values whenever we change the spreadsheet. Some times this may slow down the speed of the work. This can be avoided by turning off the calculation by using the menu bar File > Options > Formulas > Automatic except for data tables > OK. We can recalculate the table again by pressing the F9 key.

3.2 Spreadsheet Implementation of the Example

In this section, we show how to resample, without replacement, from given data, generate 1000 resamples, and calculate the F -statistic values assuming data come from a single population. *Figure 3.2* shows the spreadsheet implementation of this example. We separate our data into three sets at random and calculate the value of the test statistic. This value is recorded in the Data Table in *Figure 3.2*.

First, we put the data in a single column with the three groups going from B3:B10 (Business majors), B11:B17 (Computer Science majors), and B18:B23 (Engineering majors). Under the null hypothesis, times for all three majors are drawn from populations with the same mean and the same standard deviation, and it will not matter how they are allocated among groups. Column D contains simple random numbers generated from the RAND function. To generate these random numbers, we write the formula = RAND() in cell D3 and copy this formula to cells D4 through D23. Column E finds the rank order of these random numbers using the RANK function to produce an ordering to resample from the original data in B3:B23. This is done by writing the formula = RANK(D3,\$D\$3:\$D\$23) in cell E3 and copying this formula to cells E4 through E23. Now the VLOOKUP function and the random ordering created in column E are used to generate three samples from the data in B3:B23 and display these resampled values in F3:F23. This is done by writing the formula = VLOOKUP(E3,\$A\$3:\$B\$23,2) in cell F3 and copying this formula to cells F4 through F23. Sum of the data values for three groups are calculated in cells B25:B27 and F25:F27 for the original data and resampled data respectively. In cells B28:B29 and F28:F29, we calculate the sum of the data values and the sum of the squares of data values for entire sample for the original and resampled data. Cells B30:B31 and F30:F31 give the sum of squares between groups (SSB) and the sum of the squares within groups (SSW). Finally, we calculate the value of the F -statistic for the original data and the resampled values in cells B32 and F32 respectively.

The values of the resampled test statistic are calculated using the Data Table. The Data Table goes down as far as you want, but in this work, we create 1000 values. *Figure 3.2* shows only a portion of the spreadsheet of this implementation. Our Data Table has the cell range H2 to I1001 which calculates 1000 values of the statistic. The top right cell I2 of the table contains the formula = F32. The left column of the table is left blank. We select the cell range H2:I1001, and use the commands Data > What-if Analysis > Data Table. In the Data Table dialog box, leave

Row input cell blank and select any empty cell (say J1) for Column input cell and click OK. This will fill the right column with 1000 values of the statistic for repeated resamples. Pressing the F9 key will recalculate a different set of values of the statistic. Since students are doing these steps themselves, they visualize each step and get a clear understanding of the resampling process.

Table 3.1 shows the summary of the Excel formulas written in the spreadsheet implementation in Figure 3.2.

Cell	Formula	Purpose
D3	=RAND()	To generate simple random numbers.
E3	=RANK(D3,\$D\$3:\$D\$23)	To create a rank ordering in values in column D.
F3	=VLOOKUP(E3,\$A\$3:\$B\$23,2)	To resample the values in column B.
B25	=SUM(B3:B10)	To compute the sum of the values for Business majors.
B26	=SUM(B11:B17)	To compute the sum of the values for Computer Science majors
B27	=SUM(B18:B23)	To compute the sum of the values for Engineering majors
B28	=SUM(B3:B23)	To compute the sum of the values in the sample.
B29	=SUMPRODUCT(B3:B23,B3:B23)	To compute the sum of the squares of the values in the sample
B30	=POWER(B25,2)/8+POWER(B26,2)/7+POWER(B27,2)/6 - POWER(B28,2)/21	To compute the sum of squares between groups (SSB).
B31	=B29-(POWER(B25,2)/8 + POWER(B26,2)/7 + POWER(B27,2)/6)	To compute the sum of squares within groups (SSW).
B32	=(B30/2)/(B31/18)	To compute the value of the F - statistic.
I2	=F32	Top cell of the Data Table. Use this value to compute the Data Table
L3	=(COUNTIF(I2:I1001,">=1.297796"))/1000	To compute the p -value. This will count how many cells in range I2:I1001 are greater than or equal to 1.297796 and divide this count by 1000 to find the proportion

Table 3.1: Excel formulas

	A	B	C	D	E	F	G	H	I	J	K	L	M
1		Original				Resample		Data Table					
2	Index	Times		Random	Index	Times			3.20951				
3	1	208		0.9092	4	180			1.16		P-value	0.297	
4	2	162		0.18528	19	146			0.55474				
5	3	240		0.35892	14	147			0.17712				
6	4	180		0.95777	2	162			0.37713				
7	5	148		0.75416	10	113			0.37965				
8	6	312		0.90762	5	148			7.25522				
9	7	176		0.8735	7	176			1.59987				
10	8	292		0.12696	20	298			0.11441				
11	9	156		0.24643	16	126			0.78645				
12	10	113		0.876	6	312			1.90691				
13	11	281		0.03976	21	392			0.27625				
14	12	128		0.64031	11	281			2.52372				
15	13	305		0.93458	3	240			0.00723				
16	14	147		0.80715	9	156			0.99758				
17	15	232		0.20703	18	363			1.16592				
18	16	126		0.98571	1	208			0.12488				
19	17	275		0.61304	12	128			0.86743				
20	18	363		0.82509	8	292			3.4172				
21	19	146		0.23214	17	275			0.26973				
22	20	298		0.31722	15	232			2.08012				
23	21	392		0.50636	13	305			0.01661				
24									0.4262				
25	T1	1718				1370			0.70363				
26	T2	1362				1870			0.97547				
27	T3	1600				1440			2.65829				
28	$\sum x$	4680				4680			3.51441				
29	$\sum x^2$	1182958				1182958			0.41701				
30	SSB	17642.02				36798.214			4.02755				
31	SSW	122344.5				103188.36			1.85904				
32	F-Ratio	1.297796				3.2095087			0.88721				
33									1.53665				

Figure 3.2: Portion of the spreadsheet showing simulated values of the statistic and the p -value

4. P -Value and Empirical Distribution

In order to evaluate whether a value of the test statistic as large as the one from the observed sample is likely, we simply compute the proportion of the 1000 resamples where the test statistic values given in the Data Table exceed the observed value which is calculated in cell B32. This is known as the approximate p -value of the test and it is the proportion of values of the test statistic, assuming data come from populations with same mean, that are as extreme or more extreme than the observed value of the test statistic. This is calculated in cell L3 in Figure 3.2 using the formula = (COUNTIF(I2:I1001, ">=1.297796"))/1000 as 0.297. The theoretical p -value based on the F distribution with 2 and 18 degrees of freedom can be calculated using the Excel formula = F.DIST.RT(1.297796,2,18,) and it is 0.2975. This value agrees with our simulated value up to three decimal places. Interpreting this p -value, we can say that it is fairly likely one would observe a value of the statistic as large as, or larger than the one produced assuming there is no difference in the population averages of the times to find their first full time job for graduates of the three majors under consideration. This relatively larger p -value provides

no sufficient evidence against the claim that “the mean time taken to find their first full time job for all graduates in these fields is the same”.

We want to point out that if we run another simulation of 1000 resamples, we get a slightly different p -value in the cell L3. We can generate a new set of resamples by pressing F9 key. The following p -values were given for ten simulations: 0.296, 0.297, 0.278, 0.285, 0.282, 0.281, 0.306, 0.301, 0.302, and 0.284.

The Excel histogram in *Figure 4.1* shows the empirical distribution of the test statistic for 1000 resamples. The shaded area represents the number of values exceeding the observed value of the statistic. The pdf curve of the F distribution with 2 and 18 degrees of freedom is overlaid on histogram. The histogram of values of the statistic suggests that they are closely modeled with an F distribution with 2 and 18 degrees of freedom. This confirms that we can use the resampling approach to perform the ANOVA test.

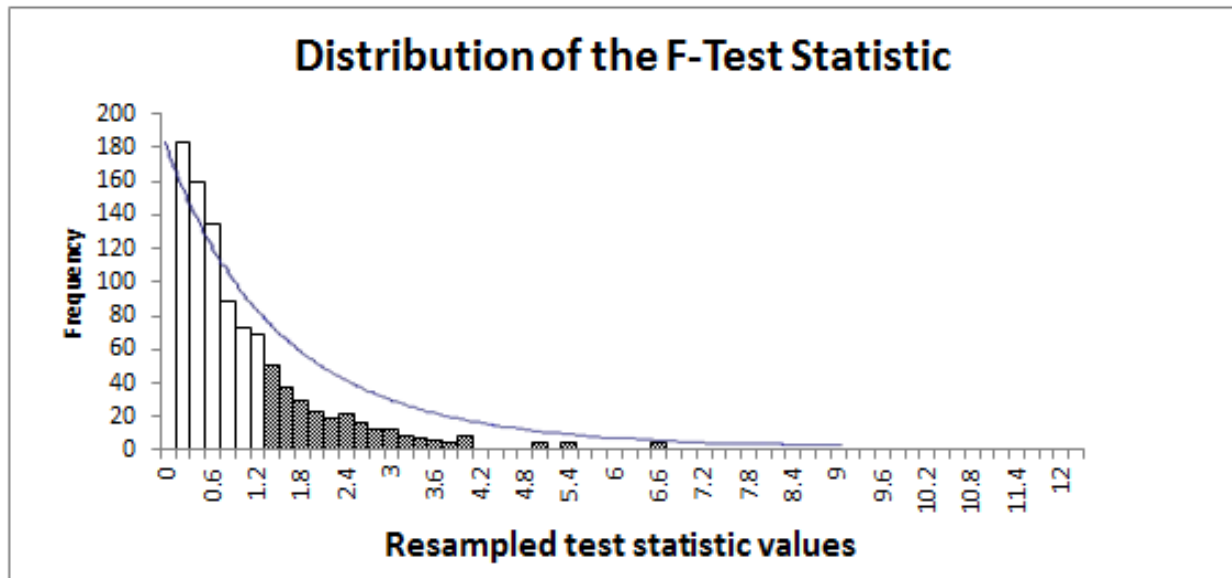


Figure 4.1: Histogram of the values of the F -test statistic

5. Comparison of Two Methods

We taught two introductory statistics sections in fall 2014, one using CSM approach with Excel described in this paper and the other by the traditional method of not using simulation. Both classes have the same instructor, same course content, same exams, same quizzes, and same assignments. Both classes were met on Mondays and Wednesdays evening schedule. The CSM class size was 28 and traditional method class size was 25. At the end of the lesson, the following survey was conducted to get an idea about students’ attitudes about the method of teaching. Each question has three answering options, namely yes, no, and no opinion.

- Q1. Method helps me to understand the concepts.
- Q2. Feeling like I am part of the discussion.
- Q3. Feeling comfortable taking part in the lesson.
- Q4. Visual representation of outcomes is useful in understanding.
- Q5. Recommend this approach to other students.

Table 5.1 gives the summary of their responses:

Question	CSM Method Class			Traditional Method Class		
	Yes	No	No Opinion	Yes	No	No Opinion
Q1	56%	24%	20%	39%	35%	26%
Q2	68%	20%	12%	35%	43%	22%
Q3	64%	24%	12%	57%	17%	26%
Q4	60%	24%	16%	57%	13%	30%
Q5	64%	16%	20%	52%	35%	13%

Table 5.1: Summary of students' responses

Question	<i>p</i> -value
Q1	0.106
Q2	0.010
Q3	0.269
Q4	0.364
Q5	0.182

Table 5.2: *p*-values

The majority of the students in the CSM class answered yes to all five questions while majority of traditional method class students answered yes to only questions 3, 4, and 5. Looking at the percentages, we observe that students are understanding concepts better and feeling more comfortable in computer simulation approach. To get a clear understanding about the percentages, we performed z-tests for comparison of proportions of students who answered yes in both teaching methods for each question. Excel does not have a specific function for comparison of two proportions. We used Excel to calculate the test statistic and *p*-value for z-test in each case. At 5% significance level, we tested the claim that the proportion of students who answered yes in CSM method was greater than the corresponding proportion in other group. The requirement of expected number of successes and failures in comparison of proportions is also met for all cases. The Table 5.2 gives the *p*-value for each case. Looking at the *p*-values, only for question two (feeling like I am part of the discussion), the proportion of students answered yes in CSM class is significantly greater than the corresponding proportion in the other class.

The quiz given in the Appendix was given to both classes after completing the ANOVA chapter. The points scale for the quiz was 0 to 100. The *Table 5.3* gives the quiz scores statistics. A dot plot for these quiz scores were created using Excel and is shown in *Figure 5.1*.

Class	<i>n</i>	Mean	Median	Std. Dev.
CSM class	28	76.85	78	14.41
Traditional method class	25	69.96	70	14.37

Table 5.3: Quiz score statistics

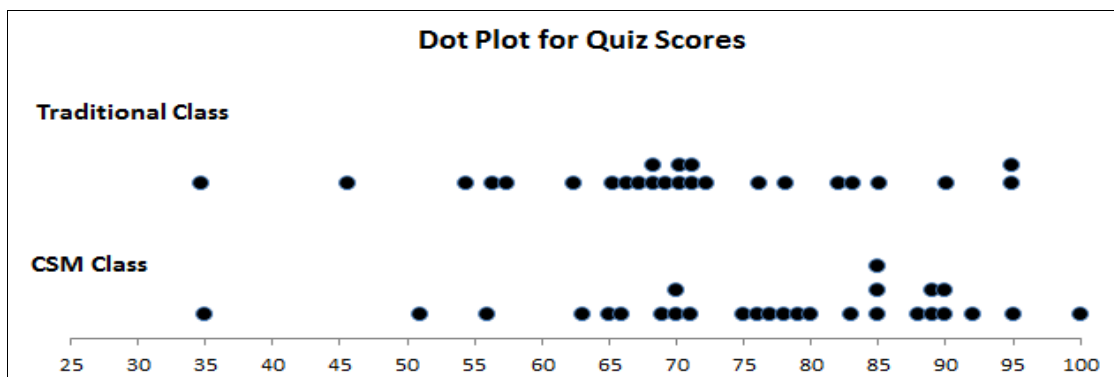


Figure 5.1: Dot Plot of quiz grades

The two sample t-test performed on quiz scores using Excel to test the hypothesis that the CSM method class performs better on average than the traditional method class. The *p*-value produced by Excel was 0.0438. This *p*-value indicates that CSM method class performs better at 0.05 significance level. We have to caution that these sample sizes for the survey and assessment statistics are not large enough to make a firm judgment on the conclusion. We plan to use larger sample sizes in the future in order to give a comprehensive survey and to make a formal assessment.

6. Conclusion

Until recent years, many statistics instructors used the traditional way of teaching which concentrated on the derivation of the test statistics whose sampling distribution can be derived mathematically. These mathematical derivations require skills beyond most students in introductory statistics classes. In contrast, the computer simulation approach described in this paper requires no mathematical skills and leads students to better understanding of the concepts. We have demonstrated how to teach ANOVA concepts using simulation. This approach of using Excel Data Tables and formulas is relatively easy for students to comprehend the concepts by implementing in a spreadsheet and visualizing the sampling distributions and *p*-values. Excel is easy to use software and many students have access to it. Our preliminary assessment suggests that computer simulation approach will enhance students' understanding of concepts and majority of students feel more comfortable using this approach. With the advancement of

technology, we can now use computer intensive methods either as an alternative to, or preferably in support of, traditional methods of teaching statistics.

References

- [1] Black, T. R. (1999). Simulation on Spreadsheets for Complex Concepts: Teaching Statistical Power as an Example, *International Journal of Mathematical Education in Science and Technology*, 30(4): 473-481.
- [2] Callaert, H. (2002). Understanding statistical misconceptions. *Proceedings of the 6th Int. Conference on the Teaching of Statistic*. Cape Town, South Africa.
- [3] Chandrakantha, L. (2014). Visualizing and Understanding Confidence Intervals and Hypothesis Testing Using Excel Simulation, *The Electronic Journal of Mathematics and Technology*, 8(3): 212-221.
- [4] Christie, D. (2004). Resampling with Excel. *Teaching Statistics*, 26(1): 9 –14.
- [5] Cobb, P. (1994). Where is the Mind? Constructivist and Sociocultural Perspectives on Mathematical Development. *Educational Researcher*, 23: 13-20.
- [6] delMas, R. C., Garfield, J., and Chance, B. L. (1999). A Model of Classroom Research in Actions: Developing Simulation Activities to Improve Students' Statistical Reasoning. *Journal of Statistics Education (Online)*, 7(3).
<http://www.amstat.org/publications/jse/secure/v7n3/delmas.cfm>.
- [7] Hagtvedt, R., Jones, G. T., and Jones, K. (2007). Pedagogical Simulation of Sampling Distributions and the Central Limit Theorem. *Teaching Statistics*, 29(3): 94 – 97.
- [8] Hagtvedt, R., Jones, G. T., and Jones, K. (2008). Teaching Confidence Intervals Using Simulation. *Teaching Statistics*, 30(2): 53 – 56.
- [9] Johnson, R. W. (2001). An Introduction to the Bootstrap. *Teaching Statistics*, 23(2): 49 – 54.
- [10] Keeler, C. and Steinhorst, K. (2001). A new approach to learning probability in the first statistics course. *Journal of Statistics Education (Online)*. 9(3).
<http://www.amstat.org/publications/jse/v9n3/keeler.html>.
- [11] Mills J. D. (2002). Using Computer Simulation Methods to Teach Statistics: Review of the Literature. *Journal of Statistical Education (Online)*, 10(1).
<http://www.amstat.org/publications/jse/v10n1/mills.html>.
- [12] Mills J. D. (2004). Learning Abstract Concepts Using Simulation, *Educational Research Quarterly*, 28(4), 18-33.
- [13] Peck R., Olsen C., and Devore J. L. (2012). *Introduction to Statistics and Data Analysis* (5th edition), Boston, MA: Cengage Learning.

- [14] Rossman, A. and Chance, B. (2011). Rossman/Chance Applet Collection.
<http://www.rossmanchance.com/applets/>.
- [15] Tempelaar, D. (2002). Modeling students learning of introductory statistics. *Proceedings of the 6th Int. Conference on the Teaching of Statistic*. Cape Town, South Africa.
- [16] Tishkovskaya, S. and Lancaster, G. (2012). Statistical Education in the 21st Century: a Review of Challenges, Teaching Innovations and Strategies for Reform, *Journal of Statistical Education (Online)*, 20(2).
<http://www.amstat.org/publications/jse/v20n2/tishkovskaya.pdf>.

Appendix

STA 250: ANOVA Quiz

1. Of the values for an ANOVA test statistic listed below, which one is likely to lead to rejecting the null hypothesis in a test for equal means?
A) 0.78 B) 2.02 C) 1.94 D) 45.1
2. When hypothesis testing involves more than two means, we use the ANOVA rather than the t -test. ANOVA stands for:
A) variation between the levels.
B) analysis of variances.
C) the estimation of the ratio of two population variances.
D) an optional variance analysis.
3. In one-way ANOVA, if the test is conducted and the null hypothesis is rejected, what does this indicate?
A) All the population means are equal.
B) At least one of the population means is different from others.
C) The normal distribution should be used instead of the F -distribution to determine the critical value for the test.
D) None of the above.
4. Which of the following statements is false regarding a one-way ANOVA?
A) Between-sample variation and within-sample variation are compared in ANOVA.
B) $SST = SSB + SSW$.
C) $F\text{-ratio} = MSB/MSW$.
D) None of the above.
5. In one-way ANOVA, the p -value is
A) the probability, assuming all means are the same, that the F - statistic will take a value at least as extreme as that actually observed.

- B) the probability, assuming means are different, that the F - statistic will take a value at least as extreme as that actually observed.
- C) the probability that all means are the same.
- D) the probability that means are different.
6. A researcher reports an F -statistic with $df = 2, 40$ from one-way ANOVA. How many treatment conditions were compared in this experiment?
 A) 2 B) 3 C) 4 D) 41
7. When conducting one-way ANOVA, the _____ the between sample variability is when compared to the within sample variability, the _____ the value of the F -statistic tend to be.
 A) smaller, larger
 B) larger, smaller
 C) larger, larger
 D) smaller, smaller
8. A psychologist studied three computer keyboard designs. Three samples of individuals were given material to type on a particular keyboard, and the number of errors committed by each participant was recorded. The data are given below:

Keyboard (Group)	Mean # of errors	Std. dev.	n
A	1	1.72	5
B	5	2.24	5
C	6	1.87	5

Are these data sufficient to conclude that there are significance differences in typing performances among the three keyboard designs? You have been asked to use one-way ANOVA technique to test this. Assume that data are three independent SRSs, one from each population and that the distribution of the errors committed is Normal. The partial ANOVA table is given below:

Source of Variation	df	SS	MS	F
Between Groups			35	
Within Groups		46		
Total				

- a) Complete the ANOVA Table.
- b) State your null and alternative hypotheses
 H_0 : _____
 H_1 : _____

- c) What are the degrees of freedom of the F distribution:
 df (between groups) = _____ df (within groups) = _____
- d) Value of F -statistic: _____
- e) p -value: _____
- f) What is your conclusion at 0.05 significance level? Interpret in terms of differences among typing performances using three keyboard designs.